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FIVE STATISTICAL TESTS TO DETERMINE THE NATURE OF THE FAILURE RATE: A SPECIAL RAM METHODOLOGY REPORT

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White Sands Missile Range, New Mexico

May 1975

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Five statistical tests are presented for verifying whether or not the failure rate is constant. Two tests are given to determine if a constant failure rate exists. The other three tests determine whether a constant, increasing, or decreasing failure rate exists.

Tables needed for three of the tests are included. Also, a numerical example is presented in which three of the tests are used for comparing results.

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I. INTRODUCTION

One of the objectives in the collection and analysis of failure time data on various pieces of equipment is to determine whether the mean time between failures (MTBF) or the mission reliability met a specified value at a desired confidence level. The confidence limits that are to be constructed on these parameters depend upon the distribution of the times between failures. In many test programs, the times between failures are assumed to come from an exponential distribution, which implies that an essentially constant failure rate exists.

It is the purpose of this report to present several statistical tests for verifying that the failure times do or do not come from an exponential distribution. If the analyst desires or finds it necessary that such a test be made, the type of test he selects will depend upon how the data are collected, the sample size, and the relative merits of the statistical tests available.

The five tests presented are for either small or large sample sizes, and their computation is simple. Two of the tests are designed specifically to determine whether a constant failure rate exists. The remaining three tests determine whether a constant, increasing, or decreasing failure rate exists.

Tables needed for three of the tests are included in Appendix A. Also, a numerical example is worked out in which three of the tests are used for comparing results.

II. SYMBOLS USED

Let $0 < X_1 < X_2 < X_3 < \ldots < X_N$ where X_i is the operating time (OT) at the ith failure. Let Y_1, Y_2, ... be the successive differences of the X's. The Y's are then the intervals between the failure time or the times between failures (TBF). In tabular form, we have:

TABLE A

i	X_i	Y_i	D_i
1	X_1	X_1	NX_1
2	X_2	$X_2 - X_1$	$(N - 1)(X_2 - X_1)$
3	X_3	$X_3 - X_2$	$(N - 2)(X_3 - X_2)$
.	.	.	.
.	.	.	.
.	.	.	.
N	X_N	$X_N - X_{N-1}$	$1 \cdot (X_N - X_{N-1})$

The D_is are called the normalized times between failures, and the figures below show the possible trends of the D_i's obtained from data (Ref 1).

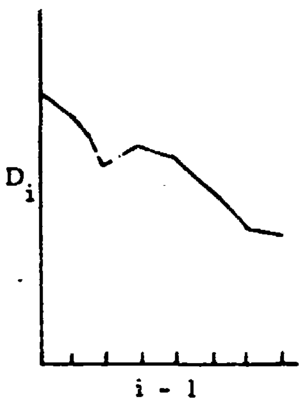


Fig. 1. The D_i's indicate an increasing failure rate.

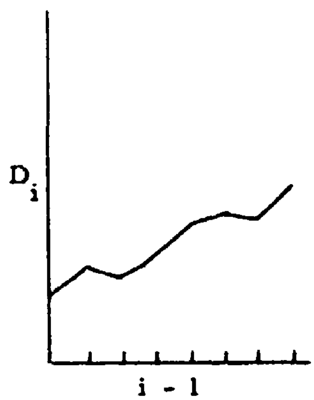


Fig. 2. The D_i's indicate a decreasing failure rate.

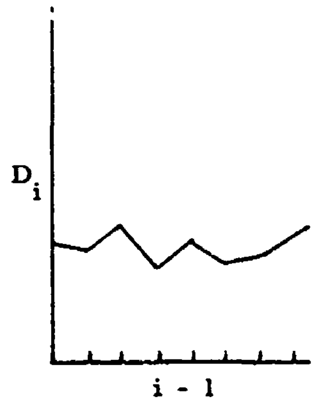


Fig. 3. The D_i's are random (no trend). This implies a constant failure rate.

A constant failure rate (CFR), λ, implies that the TBFs (y) come from an exponential distribution whose density function is $\lambda e^{-\lambda y}$, where $\lambda = \frac{1}{\theta}$, θ being the MTBF. Another failure time distribution is the Weibull, whose density function is

$$(\beta/\eta)\ (y/\eta)^{\beta - 1}\ e^{-(y/\eta)^{\beta}}$$

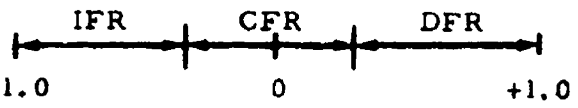
where β is the shape parameter and η is the scale parameter.

If $\beta = 1.0$, this density function reduces to the exponential. A $\beta < 1.0$ implies a decreasing failure rate (DFR), and a $\beta > 1.0$ implies an increasing failure rate (IFR). More will be said about the Weibull distribution in a later section.

III. KENDALL'S RANK CORRELATION COEFFICIENT, OR THE TAU STATISTIC

We want a statistical test to determine from a set of data (the OT or TBF) whether a CFR, IFR, or a DFR exists. Graphs such as those shown in Figures 1 through 3 may indicate a definite trend or no trend at all. In any case, fluctuations will appear in such a graph which may or may not mask a significant trend. An analytical test is then necessary for one to come to a firmer conclusion.

Kendall's tau statistic (Ref 2 and 3) tests the hypothesis that the average or expected value of tau is zero. If this hypothesis of randomness is accepted, we have evidence that a CFR exists. A calculated value of tau near -1.00 indicates an IFR; a calculated value of tau near +1.00 indicates a DFR. Graphically, we have



The formula for tau is

$$\tau = \frac{4k}{N(N-1)} - 1, \quad -1 < \tau < +1 \qquad (1)$$

where k is the number of times a y rank is followed by a larger y rank. That is, count the number y_i's (the TBF) following y_1 that exceed y_1; then count the number of y_i's following y_2 that exceed y_2. Continue this counting process through y_{N-1}, and add the number of such counts. This total is k.* (See Table A for the format of data.)

*For a computerized output, k is the number of interchanges in a simple bubble sort ranking.

A test on tau, that is, a test on $E(\tau) = 0$, is equivalent to a test on the following value of S_N, calculated from the formula

$$S_N = 2k - \frac{N(N-1)}{2} \qquad (2)$$

The value of S_N could be either positive or negative; hence, if Prob $(S \geq |S_N|) \leq \frac{\alpha}{2}$, where α is the significance level of the test, then the hypothesis that $E(\tau) = 0$ is rejected. Depending upon whether τ is negative or positive, an IFR or DFR is declared to exist.

Table 1, Appendix A, gives values of Prob $(S \geq S_N)$ for $4 \leq N \leq 10$. If $N > 10$, the normal theory approximation can be used for the test. Compute

$$z = \frac{\tau}{\frac{1}{3}\sqrt{\frac{2(2N+5)}{N(N-1)}}}$$

where z is the standard normal variable. If Prob $(Z \leq z) \leq \alpha$ when $z < 0$, reject the hypothesis of a CFR. If Prob $(Z \geq z) \leq 1 - \alpha$ when $z > 0$, also reject this hypothesis.

TABLE B

i	X_i	Y_i	D_i
1	18	18	126
2	33	15	90
3	52	19	95
4	59	7	28
5	62	3	9
6	67	5	10
7	68	1	1

Example: let

$$\alpha = 0.10$$

$$k = 1 + 1 + 0 + 0 + 1 + 0 = 3$$

$$\tau = \frac{12}{42} - 1 = -0.714$$

$$S_7 = 6 - \frac{42}{2} = -15$$

Since Prob $(S \geq |-15|) = 0.015 < \frac{\alpha}{2} = 0.05$ (Table 1), we reject the hypothesis that a CFR exists, and since τ is negative, we declare that an IFR exists. The D_i's in the above table exhibit a decreasing trend.

IV. THE PROSCHAN-PYKE TEST

An alternative test, using the D_i's above, is the Proschan-Pyke test (Ref 1). The sample size can vary from 3 to 30.

TABLE C

i	D_i	$(i - 1)\ D_{N + 1 - i}$
1	126	0
2	90	$10 = 1\ D_6$
3	95	$18 = 2\ D_5$
4	28	$84 = 3\ D_4$
5	9	$380 = 4\ D_3$
6	10	$450 = 5\ D_2$
7	1	$756 = 6\ D_1$
	$\sum = 359$	$\sum = 1698$

The hypothesis we are testing is again that the TBFs are exponentially distributed (a CFR exists). The alternative hypothesis is that an IFR exists. Compute (for $N = 7$)

$$V_N = \frac{\sum_{1}^{N} (i - 1)\ D_{N - i + 1}}{\sum_{1}^{N} D_i} = \frac{1698}{359} = 4.730 \qquad (4)$$

In Table 2 (Appendix A) for $N = 7$, $\alpha = 0.10$, the tabulated critical value is 3.917. Since the computed value of V_N exceeds 3.917, we reject the hypothesis of a CFR and accept the alternative hypothesis.*

The Proschan-Pyke test is also called a nonparametric or distribution free test. No assumption is made about the distribution of the TBF except that it is continuous. The equipment, when replaced on test after repair, is also assumed to be in as good as new condition.

*Reference 1 also gives a method for determining whether a DFR exists when V_N is much less than the tabular value.

V. THE W-TEST

The previous tests required no calculation of the MTBF or the variance of the TBF. The following W-test (Ref 4) uses these values. The sample size is from 7 to 35.

Let s^2 = the sample variance of the TBF, and $\bar{y}$ = the calculated MTBF. The test statistic is

$$W = \frac{(N - 1)s^2}{\left(\sum_1^N y_i\right)^2} \tag{5}$$

From the data in the sample,

$$(N - 1)s^2 = 333.43$$

and

$$\left(\sum_1^N y_i\right)^2 = 4624$$

Hence, W = 0.0721. This computed value is barely within the interval (0.033, 0.225). See Table 3 (App A) for N = 7, α = 0.10. Hence, it is still doubtful whether a CFR exists.

A preliminary statistic to compute is the square of the coefficient of variation (CV):

$$(CV)^2 = \frac{s^2}{\bar{y}^2} \tag{6}$$

If $s^2/\bar{y}^2 \simeq 1.00$, a CFR is suspected.

If $s^2/\bar{y}^2 < 1.00$, an IFR is suspected.

If $s^2/\bar{y}^2 > 1.00$, a DFR is suspected.

In the above example, $s^2/\bar{y}^2 = 0.59$. For a constant failure rate to exist, $\sigma^2 = \theta^2$ theoretically. Here θ = the population MTBF and σ^2 = the population variance of the times between failure.

VI. A STATISTICAL TEST INVOLVING THE WEIBULL DISTRIBUTION

If a fixed time test is considered, that is, the total operating time of the equipment is fixed in advance of testing, a statistical test is available to determine whether β, the shape parameter of the Weibull distribution (Ref 5), is one, less than one, or greater than one. An estimate of β, $\hat{\beta}$, is

$$\hat{\beta} = \frac{1}{\ln T - \frac{1}{N}\sum_{1}^{N} \ln X_i} \tag{7}$$

where T is the fixed time and $X_1 < X_2 < X_3 < \ldots < X_N \leq T$.

The 100 $(1 - \alpha)$% confidence limits on β are

$$UCL = \hat{\beta}\left[\chi^2_{1-\frac{\alpha}{2}}(2N)/2N\right] \tag{8}$$

$$LCL = \hat{\beta}\left[\chi^2_{\frac{\alpha}{2}}(2N)/2N\right] \tag{9}$$

where $\chi^2_P(2N)$ = the fractile value of chi-square at the P percent probability level with 2N degrees of freedom.

If these confidence limits span 1.00 ($\hat{\beta}$, of course, lying within this interval), then a CFR exists. If the limits are less than 1.00, a DFR exists. If the limits exceed 1.00, an IFR exists. This method is attributed to Dr. Larry Crow of the US Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, Maryland (Ref 5).

VII. THE ACCEPTANCE INTERVAL TEST

For a sample size of seven or more, the following acceptance interval test (Ref 6) is useful. Again, let the operating times at the ith failure be

$$X_1 \leq X_2 \leq X_3 \leq \ldots \leq X_N \leq T$$

and their sum be

$$X = \sum_{1}^{N} X_i \tag{10}$$

If X lies within the interval

$$\frac{NT}{2} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{NT^2}{12}} \tag{11}$$

then a CFR is accepted with $100(1 - \alpha)\%$ probability. The factor $z_{1-\frac{\alpha}{2}}$ is the standard normal variable. For $\alpha = 0.05$, $z_{0.975} = 1.96$. If $\alpha = 0.10$, $z_{0.95} = 1.645$. The acceptance interval is, therefore, based on a normal distribution approximation.

VIII. REMARKS

These five statistical tests are but a few of many to determine the nature of the failure rate. If a test-to-failure of N items is conducted with no repair or replacement of failed components, the method of matching moments test (Ref 7, p F-24) is used to find estimates of the Weibull distribution parameters if the failure times are Weibull distributed. There are also tests to help determine if an abnormally early or late failure occurred in a life test. If the sample size of failure times is very large, chi-square tests are available.

Graphical tests to determine if a CFR is likely to exist have been excluded, not because they are sometimes inadequate, but because sharper analytical tests are available. The Kolmogorov-Smirnov test (K-S test) for a CFR is also excluded because it is believed the W-test is a more powerful one, especially for small sample sizes.

All failure time data are not exponentially distributed. The tests presented here are useful techniques for helping the data analyst determine whether or not the assumption made on the failure-time distribution is valid.

APPENDIX A. TABLES

TABLE 1*

THE PROBABILITY THAT $S \geq S_N$, $4 \leq N \leq 10$

S	N = 4	N = 5	N = 8	N = 9	S	N = 6	N = 7	N = 10
0	0.625	0.592	0.548	0.540	1	0.500	0.500	0.500
2	0.375	0.408	0.452	0.460	3	0.360	0.386	0.431
4	0.167	0.242	0.360	0.381	5	0.235	0.281	0.364
6	0.042	0.117	0.274	0.306	7	0.136	0.191	0.300
8		0.042	0.199	0.238	9	0.068	0.119	0.242
10		$0.0^{2}83$	0.138	0.179	11	0.028	0.068	0.190
12			0.089	0.130	13	$0.0^{2}83$	0.035	0.146
14			0.054	0.090	15	$0.0^{2}14$	0.015	0.108
16			0.031	0.060	17		$0.0^{2}54$	0.078
18			0.016	0.038	19		$0.0^{2}14$	0.054
20			$0.0^{2}71$	0.022	21		$0.0^{3}20$	0.036
22			$0.0^{2}28$	0.012	23			0.023
24			$0.0^{3}87$	$0.0^{2}63$	25			0.014
26				$0.0^{2}29$	27			$0.0^{2}83$
28				$0.0^{2}12$	29			$0.0^{2}46$
					31			$0.0^{2}23$
					33			$0.0^{2}11$
					35			$0.0^{3}47$

*Reproduced from The Advanced Theory of Statistics by M. G. Kendall, Volume I, 5th Edition, 1952, by permission of the publishers, Charles Griffin & Co., Ltd., London & High Wycombe.

TABLE 2*

CRITICAL VALUES FOR V_N, $3 \leq N \leq 30$

N	α = 0.05	α = 0.10
3	1.684	1.553
4	2.331	2.157
5	2.953	2.753
6	3.565	3.339
7	4.166	3.917
8	4.759	4.469
9	5.346	5.056
10	5.927	5.619
11	6.504	6.178
12	7.077	6.735
13	7.647	7.289
14	8.212	7.834
15	8.777	8.385
16	9.339	8.933
17	9.899	9.480
18	10.458	10.026
19	11.015	10.570
20	11.570	11.113
21	12.124	11.655
22	12.676	12.196
23	13.227	12.736
24	13.777	13.275
25	14.326	13.813
26	14.874	14.350
27	15.421	14.887
28	15.967	15.423
29	16.513	15.958
30	17.057	16.493

*Reproduced from Statistical Inference Under Order Restrictions, by R. E. Barlow, et al., 1972, with the permission of John Wiley & Sons, New York, through N = 13. The remainder of the table was generated at WSMR.

TABLE 3*

NINETY AND 95 PERCENT INTERVALS FOR THE W-TEST, 7 ≤ N ≤ 35

N	90% Interval Lower Point	90% Interval Upper Point	95% Interval Lower Point	95% Interval Upper Point
7	0.033	0.225	0.025	0.260
8	0.032	0.200	0.025	0.230
9	0.031	0.177	0.025	0.205
10	0.030	0.159	0.025	0.184
11	0.030	0.145	0.025	0.166
12	0.029	0.134	0.025	0.153
13	0.028	0.124	0.024	0.140
14	0.027	0.115	0.024	0.128
15	0.026	0.106	0.024	0.119
16	0.025	0.098	0.023	0.113
17	0.024	0.093	0.023	0.107
18	0.024	0.087	0.022	0.101
19	0.023	0.083	0.022	0.096
20	0.023	0.077	0.021	0.090
21	0.022	0.074	0.020	0.085
22	0.022	0.069	0.020	0.080
23	0.021	0.065	0.019	0.075
24	0.021	0.062	0.019	0.069
25	0.020	0.058	0.018	0.065
26	0.020	0.056	0.018	0.062
27	0.020	0.054	0.017	0.058
28	0.019	0.052	0.017	0.056
29	0.019	0.050	0.016	0.054
30	0.019	0.048	0.016	0.053
31	0.018	0.047	0.016	0.051
32	0.018	0.045	0.015	0.050
33	0.018	0.044	0.015	0.048
34	0.017	0.043	0.014	0.046
35	0.017	0.041	0.014	0.045

*Reproduced from Statistical Models in Engineering by G. J. Hahn and S. S. Shapiro, 1967, with the permission of John Wiley & Sons, New York.

APPENDIX B. THE WEIBULL DISTRIBUTION

There are numerous methods for obtaining an estimate of the shape parameter β. No single, general, best estimate is available that requires little computation. A least-squares estimate requires a table of median ranks and a double logarithmic transformation.

The simplest formula for $\hat{\beta}$ is

$$\hat{\beta} = \frac{1.2825}{s\,(\ln y)}$$

where $s\,(\ln y)$ equals the standard deviation of the log-transformed times between failure, y. This is Menon's formula, and although it is biased compared to a least-squares estimate, the value obtained for $\hat{\beta}$ is still a good indicator of whether or not β is close to 1.00.

There is a special Weibull distribution graph paper that enables one to plot the ranked TBF versus a computed ordinate value. If these plotted points cluster closely about a straight line, then the TBFs are considered to be reasonably well Weibull distributed, and an estimate of β and η (the scale parameter) can be obtained. However, these graphical values are not very accurate because of the subjective way in which the straight line is drawn; hence, the β estimate is not recommended if confidence limits on β are desired.

APPENDIX C. REFERENCES

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